UCSC AMS 7 **Quiz 3 – Solutions** **Summer Session II, 2016**

1. A simple random sample of 900 recent college graduates was surveyed, and it was found that 171 of them smoke tobacco.

(a) (5 pts) Construct a 95% confidence interval for the ***percentage*** of all (recent) college graduates who smoke tobacco.

*The confidence interval for **proportion** has the form $\hat{p} \pm E$, where $\hat{p}$ = sample **proportion** of smokers and E = margin of error (for 95% confidence).*

(i) $\hat{p} = \frac{171}{900} = 0.19$

(ii) $E = z_{0.025} \cdot \sqrt{\frac{0.19 \times 0.81}{900}} \approx 1.96 \cdot 0.0130767 \approx 0.0256.$

(iii) Confidence interval: $0.19 \pm 0.0256 = 19\% \pm 2.56\%.$

Comment: *We use the critical z-value ($z_{0.025} = 1.96$) when computing the margin of error because $\hat{p}$ follows a normal distribution.*

(b) (5 pts) Use this data to test the hypothesis that recent college graduates smoke tobacco at a lower rate than the 27% rate of the general population. Use a 1% significance level and...

i. State clearly what the null and alternative hypotheses are.

If p is the rate at which recent college graduates smoke, then the null hypothesis says that recent college grads smoke tobacco at the same rate as the general population and the alternative hypothesis says that they smoke at a lower rate. I.e.,

$$H_0 : p = 0.27 \text{ and } H_a : p < 0.27.$$

ii. Find the test statistic.

$$z^* = \frac{\hat{p} - p_0}{\sqrt{(p_0 \times q_0)/n}} = \frac{0.19 - 0.27}{(0.27 \cdot 0.73)/900} \approx -5.4$$

where $p_0 = 0.27$ is the null hypothetical proportion and the test statistic for population proportion follows the normal distribution.

iii. Find the critical value and state what the rejection region is (traditional test).

*The critical value for $\alpha = 0.01$ and a **left-tailed** test is $-z_{0.01} \approx -2.33$ and the rejection region is $z^* < -2.33$. Since $-5.4 < -2.33$ we reject H_0.*

iv. Find the observed P-value.

$P = P(z < z^*) = P(z < -5.4) < 0.0001$. *Since* $0.0001 < 0.01$, *we reject* H_0 *(again).*

v. State your conclusion.

"There is sufficient sample evidence (at the 0.05 significance level) to support the claim that recent college graduates smoke at a lower rate than the general population".

2. A simple random sample of $n = 16$ cereal boxes is selected and the sugar content, measured in *grams of sugar per* 1 *gram of cereal*, is recorded. The data yielded the following summary statistics: $\overline{x} = 0.31$ and $s = 0.19$. You may assume that the sugar content of boxed cereal is in general (approximately) normally distributed.

(a) (5 pts) Find a 95% confidence interval for the mean sugar content of all boxed cereals.

The confidence interval for ***mean sugar content*** *has the form* $\overline{x} \pm E$, *where* $\overline{x} =$ ***sample mean*** *sugar content and* $E =$ *margin of error (for* 95% *confidence).*

(i) $\overline{x} = 0.31$

(ii) Since the sugar content of boxed cereal follows a normal distribution and we are using the sample standard deviation, the random variable $\frac{\overline{x}-\mu}{s/\sqrt{n}}$ *(with sample size* $n = 16$*) follows the t-distribution with* 15 *degrees of freedom. Therefore we use the critical t-value (for* 15 *d.f.) when computing the margin of error, so*

$$E = t_{0.025} \cdot \frac{s}{\sqrt{n}} = 2.131 \cdot \frac{0.19}{4} \approx 0.101.$$

(iii) Confidence interval for mean sugar content in boxed cereal: 0.31 ± 0.101.

(b) (5 pts) Use this data to test the hypothesis that the mean sugar content of boxed cereal is *greater* than 0.20 grams per 1 gram of cereal. Use a 1% significance level and...

i. State clearly what the null and alternative hypotheses are.

The null hypothesis says that the mean sugar content of boxed cereal (μ) *is* ***equal to*** 0.2 *and the null hypothesis says that it is* ***greater than*** 0.2. *I.e.,*

$$H_0 : \mu = 0.2 \ \ \textit{and} \ \ H_a : \mu > 0.2.$$

ii. Find the test statistic.

$$t^* = \frac{\overline{x} - \mu_0}{s/\sqrt{n}} = \frac{0.31 - 0.2}{0.19/4} \approx 2.316.$$

iii. Find the critical value and state what the rejection region is.

The test statistic follows the t-distribution with 15 *d.f., so the critical value for* $\alpha = 0.01$ *(and a right-tailed test) is* $t_{0.01} = 2.602$ *and the rejection region is* $t^* > 2.602$. *In this case, we* ***fail to reject*** H_0 *because* $2.316 < 2.602$.

iv. *Estimate* the observed P-value.

Since $t^* = 2.316$ *falls between the critical values for* $\alpha = 0.01$ *and* $\alpha = 0.025$ *(i.e.,* $2.131 < 2.316 < 2.602$*), we estimate that* $0.01 < P < 0.025$.

v. State your conclusion.

"There is ***not*** *enough sample evidence (at the* 0.01 *significance level) to support the claim that the mean sugar content of boxed cereal is greater than* 0.2*".*